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**USAF AIRCRAFT TAKEOFF LENGTH DISTANCES  
AND CLIMBOUT PROFILES**

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The Air Force Weapons Laboratory in cooperation with the Argonne National Laboratory has developed a computerized Air Quality Assessment Model (AQAM) which will be used to assess the impact of airbase activity on Ambient Air Quality within the airbase and in surrounding areas. The AQAM has the capability of precisely simulating the aerial and ground operating cycle of each active USAF aircraft. An important component of this operating cycle is the takeoff runway roll distance. The runway roll is associated (over)		

with high engine power settings and correspondingly high pollution emissions. This report outlines the procedure for utilizing least-squares curve fitting techniques to develop equations to accurately define the runway roll distance for each USAF aircraft in given meteorological conditions. The report presents the equations for nearly all USAF aircraft and their associated takeoff runway roll under a typical meteorological condition. In addition, the takeoff climb angles for each aircraft under minimum and maximum weight load conditions were calculated and are presented in a table.

This final report was prepared by the Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico under Job Order 21033A11. Captain Dennis F. Naugle (DEE) was the Laboratory Project Officer-in-Charge.

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## PREFACE

The author wishes to express his appreciation to Capt John Shipe of the AFWL/WE for technical assistance in the development of techniques to determine the takeoff length equations and to Capt Dennis Naugle of the AFWL/DE for editorial comments and suggestions.

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## SECTION I

### INTRODUCTION

The United States Air Force in a contractual effort with Argonne National Laboratory has developed a computerized Air Quality Assessment Model (AQAM) which will be used to more accurately assess the impact of airbase activity on the quality of the ambient air. Although the AQAM can handle most emission sources, its most noted feature is detailed algorithms to describe aircraft aerial and ground operations. Each aircraft type is treated specifically through accurate definition of engine emission factors, landing and takeoff (LTO) cycle times, takeoff and approach profiles, and takeoff runway roll distances. One of the most variable components of all aircraft operations is the runway roll distance. This distance varies greatly from aircraft to aircraft and with changing meteorological conditions. For example, at ambient temperature of 60°F, pressure altitude of 3000 feet and a headwind of 3 knots, the KC-135 with a gross weight of 220,000 pounds has a runway roll of about 6300 feet as compared to the C-141 with the same gross weight which has a runway roll of about 2600 feet. In addition, the runway roll for the KC-135 can decrease by as much as 60 percent due to an increase in headwind velocity of 20 knots. Similar variations are exhibited by most USAF aircraft.

The importance of precisely modeling this runway roll distance lies in the fact that, in most cases, aircraft in this mode are in the highest engine power setting (afterburner) thus producing high emissions for some pollutants. These pollutants can be precisely distributed along the runway by knowing the time each aircraft is in the runway roll mode, the emission factor for that engine operating mode, and the number of operations. The time factor is computed by multiplying the takeoff speed by the runway roll distance. If the runway roll distance is accurately defined, this computational method will assure accurate modeling of the downwind pollutant dispersion.



This method of modeling runway roll emissions is employed by the AQAM. Equations to calculate the runway roll distance were developed from charts of performance characteristics and put into analytical form for use in the computer model. The charts are contained in USAF Flight Manuals, USAF Technical Order 1-1 series, and give information concerning the runway roll distance of a particular aircraft under given meteorological conditions of air temperature, pressure altitude, headwind velocity and the gross aircraft weight. Least-squares curve fitting techniques were applied to the data in these charts to determine an equation of four variables for each active aircraft in the Air Force.

The procedure used to develop the equations is outlined in the following section with a detailed example in appendix C. The equations themselves are presented in appendix A with some typical runway roll distances in appendix B.



## SECTION II

### PROCEDURE

The performance characteristics of all USAF aircraft can be found within USAF Technical Order 1-1 series commonly referred to as "1-1 Flight Manuals." The information contained within the 1-1 Flight Manual is in the form of tables and graphs and concerns the performance of an aircraft during both aerial and ground operations.

#### 1. MANUAL USE OF 1-1 FLIGHT MANUAL

a. Graphs such as those shown in figure 1, 2, and 3 are similar to ones found in the 1-1 Flight Manual. These graphs are used to determine the runway roll distance for a given set of meteorological and aircraft weight load conditions.

b. The graphs in figure 1, 2, and 3 can be used to manually determine the runway roll distance in the following manner:

(1) Determine the takeoff factor from figure 1 for given outside air temperature and pressure altitude. For example, an outside air temperature of 72° and a pressure altitude of 6000 feet yields a takeoff factor of 5.6.

(2) Determine the ground run from figure 2 for the takeoff factor found in step (1) and the gross weight of the aircraft. For example, if the takeoff factor is 5.6 and the gross weight of the aircraft is 110,000 pounds the ground run is 2750 feet.

(3) Determine the true or final ground run from figure 3 for a given headwind velocity and the ground run found in step (2). For example, with a ground run of 2750 feet and a 20-knot headwind the final ground run distance is 2200 feet.

#### 2. USE OF CHARTS TO DEVELOP TAKEOFF LENGTH EQUATIONS

Least-squares curve fitting techniques can be applied to data taken from a curve on a graph of two variables (see example in appendix C).

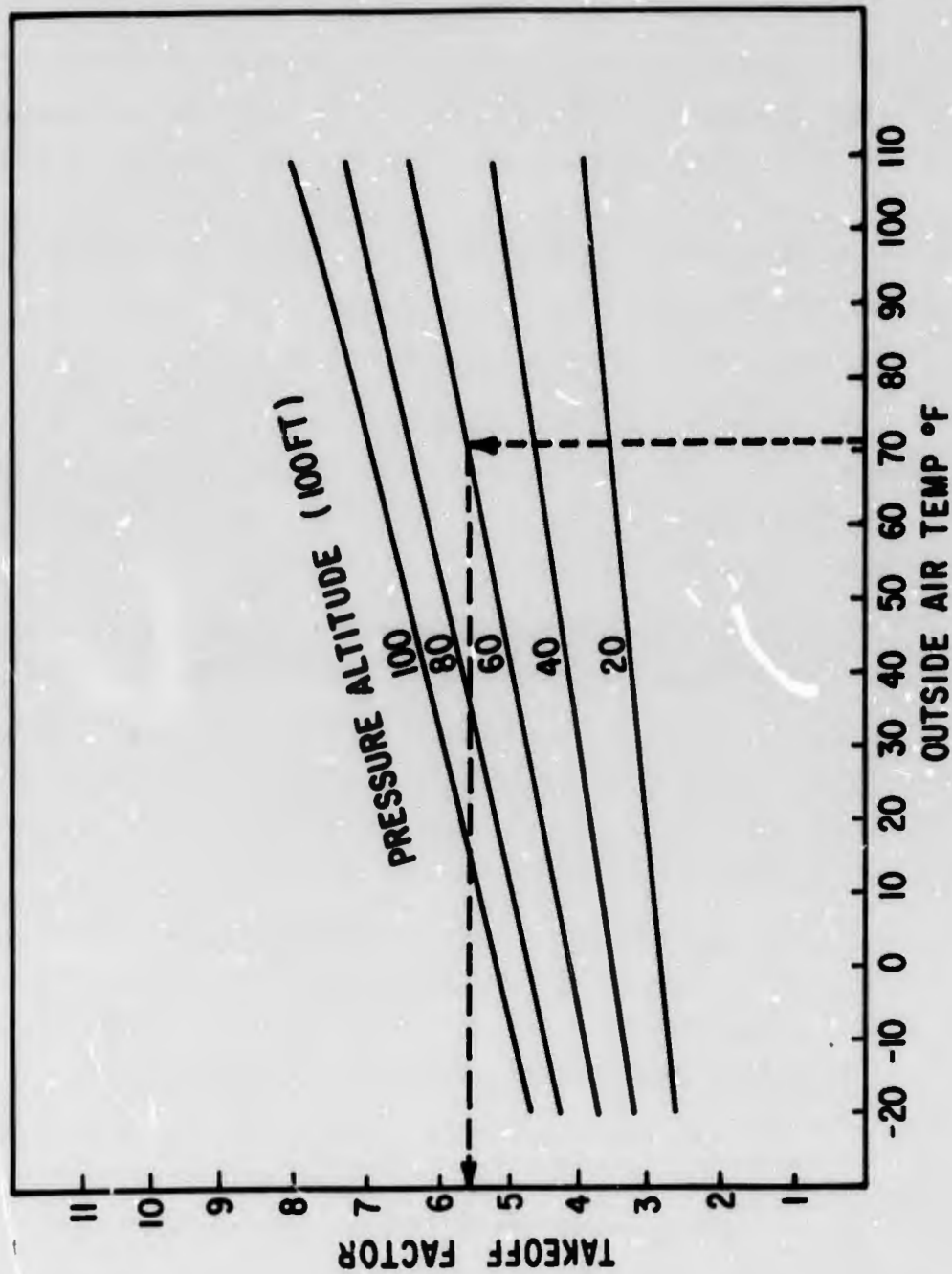


Figure 1. Takeoff Factor as a function of Temperature and Pressure Altitude

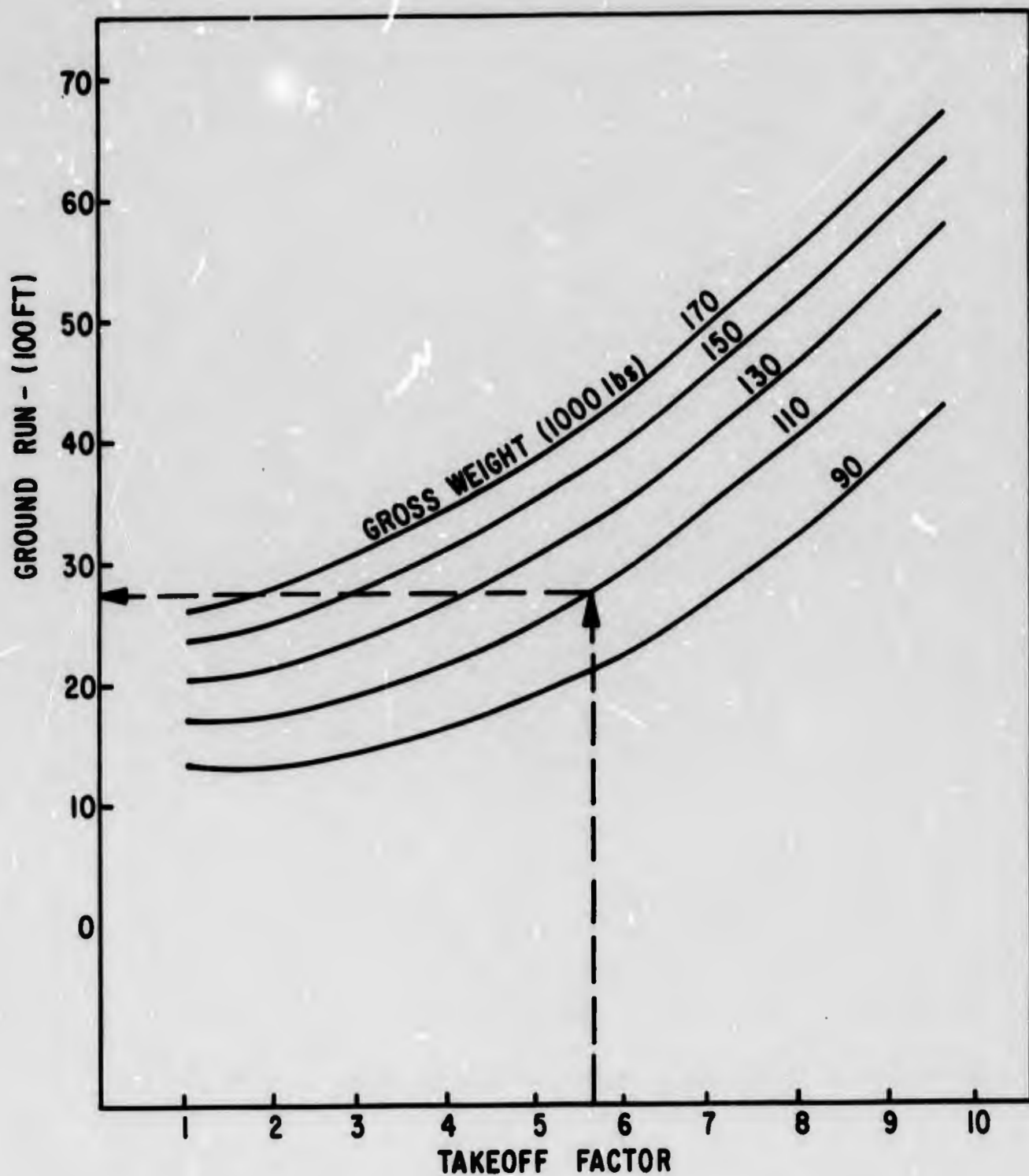


Figure 2. Ground Run as a function of Takeoff Factor and Aircraft Gross Weight

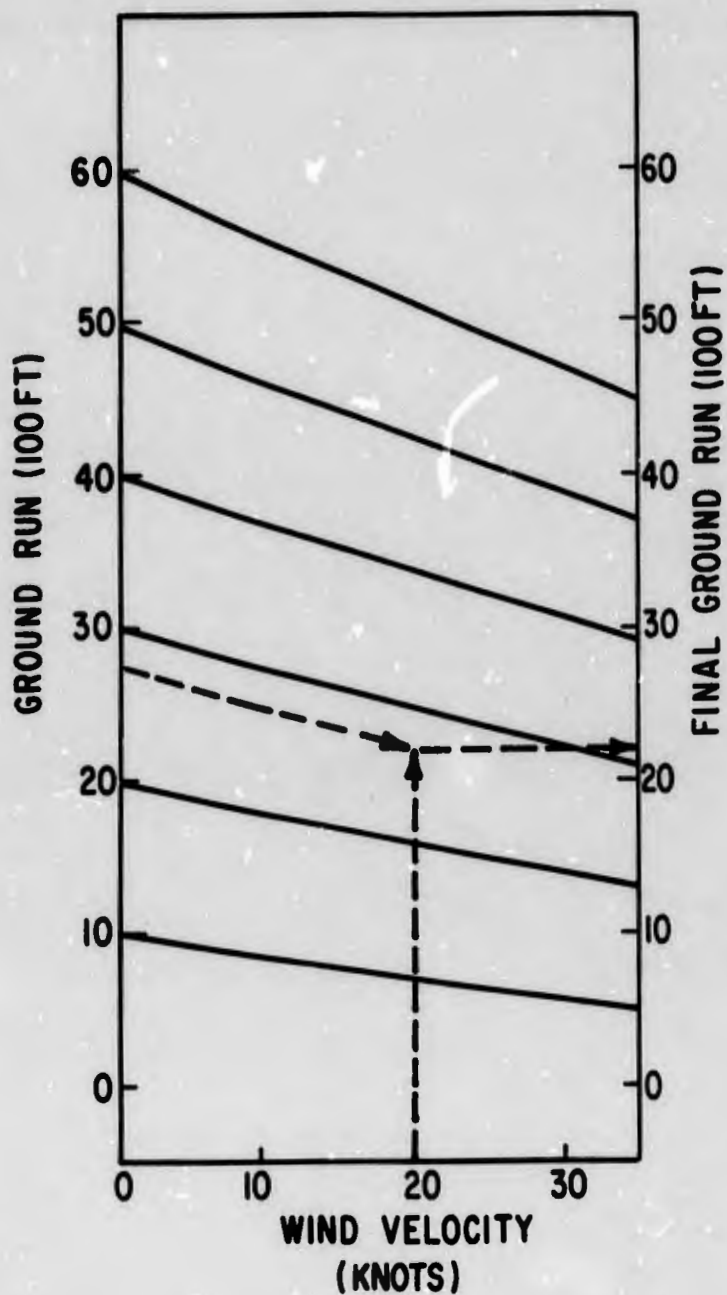


Figure 3. Final Ground Run as a function of Ground Run and Headwind Velocity

The dependence of one variable on another can be mathematically determined and, if linear, expressed in the form

$$Y = m_1X + m_0$$

where  $Y$  = the dependent variable

$X$  = the independent variable

$m_1$  = the first degree coefficient

$m_0$  = a constant

Figure 1 shows straight lines, therefore, implying a linear relationship between takeoff factor (TOF) and temperature (T). An equation can be determined for a given pressure altitude (PA) and takes the form

$$\text{TOF} = a_1T + a_0$$

where  $a_1$  = the first degree coefficient

$a_0$  = a constant

Equations can be determined for five different pressure altitudes which result in a group of equations in the form

$$(^1\text{PA}) \text{ TOF}_1 = ^1a_1T + ^1a_0$$

$$(^2\text{PA}) \text{ TOF}_2 = ^2a_1T + ^2a_0$$

⋮

$$(^5\text{PA}) \text{ TOF}_5 = ^5a_1T + ^5a_0$$

It is possible to relate the coefficients  $a_1$  and  $a_0$  to pressure altitude by equations in the form

$$a_1 = ^2b_2\text{PA}^2 + ^2b_1\text{PA} + ^2b_0$$

$$a_0 = ^1b_2\text{PA}^2 + ^1b_1\text{PA} + ^1b_0$$

where  $b_2$  = the second degree coefficient

$b_1$  = the first degree coefficient

$b_0$  = a constant

By substitution, the resulting equation for determining takeoff factor as a function of pressure altitude and temperature is

$$TOF = ({}^2b_2PA^2 + {}^2b_1PA + {}^2b_0)T + ({}^1b_2PA^2 + {}^1b_1PA + {}^1b_0)$$

A similar procedure is used to develop a general equation from the curves in figure 2. However, since the relationship between takeoff factor and ground run (GR) is not linear, the equations will be at least second order and, since their slopes appear to be non-linear with respect to gross weight (GW), the coefficients will also be at least a second order function of gross weight. The developed equations are in the form

$$GR = c_2TOF^2 + c_1TOF + c_0$$

where  $c_2$  = the second degree coefficient

$c_1$  = the first degree coefficient

$c_0$  = a constant

As before, an equation for ground run is calculated for at least five gross aircraft weights and their coefficients mathematically expressed as a function of gross weight. The resultant polynomial is in the form

$$\begin{aligned} GR = & ({}^3d_2GW^2 + {}^3d_1GW + {}^3d_0)TOF^2 \\ & + ({}^2d_2GW^2 + {}^2d_1GW + {}^2d_0)TOF \\ & + ({}^1d_2GW^2 + {}^1d_1GW + {}^1d_0) \end{aligned}$$

where  $d_2$  = the second order coefficient

$d_1$  = the first order coefficient

$d_0$  = a constant

The relationship between final ground run (FGR) and headwind velocity (WS) in figure 3 is linear. The slope of the lines change linearly with ground run (GR) and thus the coefficients can be expressed as a linear function of ground run. The equation is expressed in the form

$$FGR = f_1WS + f_0$$

where  $f_1$  = the first order coefficient

$f_0$  = a constant

It is clear that GR can be substituted for  $f_0$  since at a wind velocity of zero the final ground run (FGR) equals the ground run. Coefficient  $f_1$  can be expressed as a linear function of WS and results in a final equation in the form

$$FGR = GR - (g_1 GR + g_0) WS$$

where  $g_1$  = the first order coefficient

$g_0$  = a constant

The complete equation is expressed in the form

$$TOF = ({}^2b_2 PA^2 + {}^2b_1 PA + {}^2b_0) T + ({}^1b_2 PA^2 + {}^1b_1 PA + {}^1b_0)$$

$$\begin{aligned} GR = & ({}^3d_2 GW^2 + {}^3d_1 GW + {}^3d_0) TOF^2 \\ & + ({}^2d_2 GW^2 + {}^2d_1 GW + {}^2d_0) TOF \\ & + ({}^1d_2 GW^2 + {}^1d_1 GW + {}^1d_0) \end{aligned}$$

$$FGR = GR - (g_1 GR + G_0) WS$$

where TOF = Takeoff Factor

PA = Pressure Altitude

T = Temperature

GR = Ground Run

GW = Gross Aircraft Weight

FGR = Final Ground Run

WS = Headwind Velocity

### 3. ACCURACY OF THE EQUATIONS

a. At the conclusion of each step in the development of the takeoff length (TOL) equations the resultant least-squares lines were tested for accuracy. Values for X were input into the equation being tested and a corresponding Y value was calculated. The calculated Y value was compared to the actual Y value as found on the graphs. This



was done for at least five X values. The equation was judged acceptable if all five of the calculated Y values were within 10 percent of the actual Y values (see appendix C). If the equation was found to be not acceptable, a higher order least-squares fit was applied to the data to obtain an equation which could meet the accuracy limit.

b. When all steps in the above procedure had been completed and the TOL equation had been assembled, the equation was tested for accuracy. Although each portion of the TOL equation was accurate to within 10 percent small amounts of error could accumulate in the calculation and yield an inaccurate TOL value. This was found to be the case in about one third of all TOL equations. This problem was corrected, in many cases, by determining a higher order least-squares fit in some portions of the TOL equation, particularly in the portions designed to calculate the first and second order coefficients. If a TOL equation was still unacceptable the equation was written into a computer program and the program put in an "on-line" mode to the computer. The numbers within the TOL equation could then be manually changed and the resulting calculated value could be quickly determined. This "trial and error" correction method was continued until the equation yielded values within 10 percent of the actual values. This method was successful in every case attempted.

#### 4. DEVIATIONS IN THE GENERAL FORM OF THE EQUATIONS

This procedure was generally used to determine all equations. However, variations in the input data made necessary some minor procedural changes and caused the final form of some equations to slightly deviate from the above.

a. For example, in determining the takeoff length equation for the OV40 aircraft it was found that the relationship between takeoff factor and temperature is not linear and the relationship between ground run and takeoff factor is linear. The relationship between the coefficients of the takeoff factor equations and pressure altitude and the relationship between the coefficients of the ground run equations

and gross weight were found to be nonlinear. These relationships caused the final form of the F-100 TOL equation to differ slightly from the general form (see appendix A).

b. The equations for the B-52 and C-5 aircraft required special forms because the graphical relationship of variables, as presented in the B-52 and C-5 1-1 Flight Manuals, is different from the relationship of variables for all other aircraft (see appendix A).

#### 5. USE OF 1-1 FLIGHT MANUALS TO DETERMINE CLIMBOUT ANGLES

a. The 1-1 Flight Manuals provide information regarding the climbout characteristics of an aircraft during times of minimum and maximum weight load conditions. The small load conditions largely determine the climbout angle, with the larger angles associated with lower weight loads and smaller angles with higher weight loads.

b. The information is generally presented in the form of graphs or tables which give the horizontal ground distance traveled to reach a specified vertical flight distance, usually 5000 feet. The climbout angle can be determined by an equation in the form

$$\theta = \arctan \frac{v}{h}$$

where  $\theta$  = climbout angle

$h$  = Horizontal distance

$v$  = Vertical flight distance

The maximum and minimum climbout angles for most USAF aircraft are presented in appendix D.

## SECTION III

### CONCLUSIONS

It was found that the takeoff length can vary greatly from one aircraft to another under the same meteorological conditions (see appendix B). For example, at a temperature of 60°F, pressure altitude of 3000 feet and headwind velocity of 3 knots, the OV-10 aircraft has a takeoff length of about 1000 feet. At the same meteorological conditions the F-100 has a takeoff length of about 7200 feet, a difference of over 600 percent. This difference is due to the combination of takeoff weight, engine characteristics and structural design. The takeoff length for each individual aircraft will vary due to changes in the meteorological conditions and takeoff weight. For example, at a temperature of 20°F, pressure altitude of 2000 feet, takeoff gross weight of 30,000 pounds and a headwind velocity of 20 knots the F-100 aircraft has a takeoff length of about 2500 feet. If the takeoff weight is increased to 40,000 pounds the takeoff length is about 5000 feet, an increase of 100 percent. If the takeoff weight is reduced to 30,000 pounds and the temperature increased to 60° the takeoff length will be 50 percent longer. Another factor causing significant takeoff length variation is changes in headwind velocity. A change in headwind velocity of 25 knots can alter the takeoff length of an aircraft by as much as 100 percent.

The change in aircraft weight load can significantly alter the takeoff climb angle (see appendix D). For example, the maximum climb angle for the C-130 under maximum weight load conditions is 2° and 15° under minimum weight load. This is a change of about 650 percent.

The fighter aircraft generally have the steepest maximum and minimum climb angles (see appendix D). The F-4 and F-101 for example, have maximum climb angles of, respectively, 47° and 39°. These same two aircraft have minimum climb angles of, respectively, 21° and 22°. The Cargo and Training aircraft show very shallow minimum climb angles and

a large average variation between maximum and minimum angles (see appendix D).

The TOL equations can accurately describe the changes in takeoff length distance for each active USAF aircraft due to variation in aircraft weight and meteorological conditions. The equations, in analytical form and the takeoff climb angles, are designed into the AQAM. Their use, by the model, add to the overall ability of precise air quality modeling.

## APPENDIX A

### TAKEOFF LENGTH EQUATIONS

Equations to define the runway roll distance for most of the active aircraft in the USAF have been determined. The units for the variables used in the equations are as follows: Temperature (T) in degrees, Pressure Altitude (PA) in hundreds of feet, Gross Weight (GW) in thousands of pounds, Ground Run (GR) and Final Ground Run (FGR) in hundreds of feet and Wind Speed (WS) in knots. The aircraft are divided into six categories; Bombers, Fighters, Attack, Cargo, Training and Observation and are presented below:

#### 1. BOMBERS

##### B-52

$$\begin{aligned} \text{TOF} &= - [2.78 - 8.5714 \times 10^{-4}(\text{PA})] + [1.82 \times 10^{-2} + 7.2857 \times 10^{-5}(\text{PA})] \text{GW} \\ \text{GR} &= [1.184 \times 10^1 - 4.2167 \times 10^{-1}(\text{T}) + 1.0 \times 10^{-2}(\text{T})^2 - 4.583 \times 10^{-5}(\text{T})^3] \\ &\quad + [4.194 + 1.7197 \times 10^{-2}(\text{T}) - 9.26018 \times 10^{-4}(\text{T})^2] \text{TOF} \\ &\quad + [1.0457 + 8.40 \times 10^{-3}(\text{T}) + 2.117 \times 10^{-4}(\text{T})^2 + 2.98 \times 10^{-7}(\text{T})^3] \text{TOF}^2 \\ \text{FGR} &= \text{GR} - [1.15 \times 10^{-1} + 9.0 \times 10^{-3}(\text{GR})] \text{WS} \end{aligned}$$

##### B-57

$$\begin{aligned} \text{TOF} &= [1.589 + 6.883 \times 10^{-3}(\text{PA}) + 1.2767 \times 10^{-4}(\text{PA})^2] \\ &\quad + [8.819 \times 10^{-3} + 1.1007 \times 10^{-4}(\text{PA}) - 3.924 \times 10^{-7}(\text{PA})^2] \text{T} \\ &\quad + [5.979 \times 10^{-5} + 3.38096 \times 10^{-7}(\text{PA}) + 8.532 \times 10^{-9}(\text{PA})^2] \text{T}^2 \\ \text{GR} &= [-13.25 + 8.75 \times 10^{-1}(\text{GW}) - 1.25 \times 10^{-2}(\text{GW})^2] \\ &\quad + [1.3925 \times 10^1 - 9.275 \times 10^{-1}(\text{GW}) + 2.125 \times 10^{-2}(\text{GW})^2] \text{TOF} \\ \text{FGR} &= \text{GR} - [1.316 \times 10^{-1} + 8.748 \times 10^{-3}(\text{GR})] \text{WS} \end{aligned}$$

S-101

$$\begin{aligned}
\text{TOF} &= [-1.06 \times 10^{-3} + 1.674 \times 10^{-2}(\text{PA}) + 8.1888 \times 10^{-5}(\text{PA})^2] \\
&+ [1.36 \times 10^{-2} + 9.592 \times 10^{-6}(\text{PA}) + 1.755 \times 10^{-6}(\text{PA})^2]T \\
&+ [5.1099 \times 10^{-5} + 1.2899 \times 10^{-6}(\text{PA}) - 6.123 \times 10^{-9}(\text{PA})^2]T^2 \\
\text{GR} &= [-1.423 \times 10^1 + 6.349998 \times 10^{-1}(\text{GW}) + 1.6667 \times 10^{-3}(\text{GW})^2] \\
&+ [6.1857 - 3.2179 \times 10^{-1}(\text{GW}) + 8.214 \times 10^{-3}(\text{GW})^2] \text{TOF} \\
\text{FGR} &= \text{GR} - [6.293 \times 10^{-2} + 7.328 \times 10^{-3}(\text{GR})] \text{WS}
\end{aligned}$$

F-102

$$\begin{aligned}
\text{TOF} &= [9.503 \times 10^{-2} + 3.313 \times 10^{-2}(\text{PA}) + 1.3666 \times 10^{-4}(\text{PA})^2] \\
&+ [2.2546 \times 10^{-2} + 1.7848 \times 10^{-4}(\text{PA}) - 4.04 \times 10^{-6}(\text{PA})^2]T \\
&+ [1.3438 \times 10^{-4} - 1.2166 \times 10^{-6}(\text{PA}) + 4.1854 \times 10^{-8}(\text{PA})^2]T^2 \\
\text{GR} &= [2.95 \times 10^1 - 2.394(\text{GW}) + 6.497 \times 10^{-2}(\text{GW})^2] \\
&+ [3.1035 + 7.52 \times 10^{-2}(\text{GW}) - 3.186 \times 10^{-3}(\text{GW})^2] \text{TOF} \\
&+ [1.2715 - 1.5535 \times 10^{-1}(\text{GW}) + 4.3889 \times 10^{-3}(\text{GW})^2] \text{TOF}^2 \\
\text{FGR} &= \text{GR} - [-9.0 \times 10^{-2} + 1.807 \times 10^{-2}(\text{GR}) - 7.143 \times 10^{-5}(\text{GR})^2] \text{WS}
\end{aligned}$$

F-104

$$\begin{aligned}
\text{TOF} &= [-3.36455 \times 10^{-3} + 5.63556 \times 10^{-2}(\text{PA})] \\
&+ [4.417 \times 10^{-2} - 2.031 \times 10^{-3}(\text{PA}) + 5.63 \times 10^{-5}(\text{PA})^2 - 3.9954 \times 10^{-7}(\text{PA})^3]T \\
&+ [-9.2 \times 10^{-5} + 2.08 \times 10^{-5}(\text{PA}) - 5.39 \times 10^{-7}(\text{PA})^2 + 3.8 \times 10^{-9}(\text{PA})^3]T^2 \\
\text{GR} &= [1.65838 - 3.069 \times 10^{-1}(\text{GW}) + 8.1363 \times 10^{-2}(\text{GW})^2] \\
&+ [-3.6111 + 3.63559 \times 10^{-1}(\text{GW})] \text{TOF} \\
&+ [7.3975 \times 10^{-1} - 8.78749 \times 10^{-2}(\text{GW}) + 3.2487 \times 10^{-3}(\text{GW})^2] \text{TOF}^2 \\
\text{FGR} &= \text{GR} - [5.0 \times 10^{-2} + 7.4 \times 10^{-3}(\text{GR})] \text{WS}
\end{aligned}$$

F-105

$$\begin{aligned}\text{TOF} &= [12.5546 - 5.7192 \times 10^{-2}(\text{PA}) + 1.3075 \times 10^{-4}(\text{PA})^2] \\ &\quad - [2.9032 \times 10^{-2} - 1.0254 \times 10^{-4}(\text{PA}) - 1.45125 \times 10^{-7}(\text{PA})^2]T \\ \text{GR}' &= [-5.14955 \times 10^1 + 2.57957(\text{GW}) - 1.4425 \times 10^{-2}(\text{GW})^2] \\ &\quad - [-1.1535 \times 10^1 + 5.915 \times 10^{-1}(\text{GW}) - 4.6828 \times 10^{-3}(\text{GW})^2]_{\text{TOF}} \\ &\quad + [-6.2285 \times 10^{-1} + 3.2375 \times 10^{-2}(\text{GW}) - 2.9056 \times 10^{-4}(\text{GW})^2]_{\text{TOF}}^2\end{aligned}$$

$$\text{GR} = \text{GR}' \times 1000$$

$$\begin{aligned}\text{FGR}' &= [3.305 \times 10^1 + 9.729 \times 10^{-1}(\text{GR}) + 2.31 \times 10^{-6}(\text{GR})^2] \\ &\quad - [8.244 + 8.3598 \times 10^{-3}(\text{GR}) - 1.44 \times 10^{-8}(\text{GR})^2]_{\text{WS}}\end{aligned}$$

$$\text{FGR} = \text{FGR}'/100$$

F-106

$$\begin{aligned}\text{TOF} &= [7.436 \times 10^{-1} + 4.29 \times 10^{-2}(\text{PA})] + [2.1276 \times 10^{-2} - 3.1116 \times 10^{-5}(\text{PA})]T \\ \text{GR} &= [1.638 \times 10^1 - 7.78 \times 10^{-1}(\text{GW}) + 2.84 \times 10^{-2}(\text{GW})^2] \\ &\quad + [3.809 - 1.947 \times 10^{-1}(\text{GW}) + 4.264 \times 10^{-3}(\text{GW})^2]_{\text{TOF}} \\ &\quad + [-1.976 \times 10^{-1} + 1.5757 \times 10^{-2}(\text{GW}) + 4.6189 \times 10^{-4}(\text{GW})^2]_{\text{TOF}}^2 \\ \text{FGR} &= \text{GR} - [8.5 \times 10^{-2} + 8.25 \times 10^{-3}(\text{GR})]_{\text{WS}}\end{aligned}$$

F-111

$$\begin{aligned}\text{TOF} &= [2.336 + 1.582 \times 10^{-2}(\text{PA}) + 1.172 \times 10^{-4}(\text{PA})^2] \\ &\quad + [5.604 \times 10^{-3} + 9.97746 \times 10^{-5}(\text{PA}) - 5.8117147 \times 10^{-7}(\text{PA})^2]T \\ &\quad + [9.19269 \times 10^{-5} - 1.34357 \times 10^{-8}(\text{PA}) + 1.61411 \times 10^{-8}(\text{PA})^2]T^2 \\ \text{GR} &= [7.7366 - 2.52997 \times 10^{-1}(\text{GW}) + 2.385 \times 10^{-3}(\text{GW})^2] \\ &\quad + [-2.1071 + 4.2586 \times 10^{-2}(\text{GW}) + 12.748 \times 10^{-4}(\text{GW})^2]_{\text{TOF}} \\ \text{FGR} &= \text{GR} - [1.0755 \times 10^{-1} + 1.4588 \times 10^{-2}(\text{GR}) - 7.94156 \times 10^{-5}(\text{GR})^2]_{\text{WS}}\end{aligned}$$



### 3. ATTACK

#### A-7

$$\begin{aligned} \text{TOF} &= [7.6859 - 1.15 \times 10^{-1}(\text{PA}) + 4.413 \times 10^{-4}(\text{PA})^2] \\ &\quad - [2.925 \times 10^{-2} - 8.1128 \times 10^{-4}(\text{PA}) + 6.999 \times 10^{-6}(\text{PA})^2] \text{T} \\ &\quad - [2.2289 \times 10^{-4} + 5.054 \times 10^{-6}(\text{PA}) - 7.57 \times 10^{-8}(\text{PA})^2] \text{T}^2 \\ \text{GR} &= [2.546 \times 10^1 - 2.3388(\text{GW}) + 1.0717 \times 10^{-1}(\text{GW})^2] \\ &\quad - [7.9095 - 6.7434 \times 10^{-1}(\text{GW}) + 2.1045 \times 10^{-2}(\text{GW})^2] \text{TOF} \\ &\quad + [6.099 \times 10^{-1} - 5.0858 \times 10^{-2}(\text{GW}) + 1.434 \times 10^{-3}(\text{GW})^2] \text{TOF}^2 \\ \text{FGR} &= \text{GR} - [1.16 \times 10^{-1} + 7.27 \times 10^{-3}(\text{GR}) - 3.64 \times 10^{-6}(\text{GR})^2] \text{WS} \end{aligned}$$

#### A-37

$$\begin{aligned} \text{TOF} &= [2.118 + 1.058 \times 10^{-2}(\text{PA}) + 1.014 \times 10^{-4}(\text{PA})^2] \\ &\quad + [2.102 \times 10^{-3} + 1.84 \times 10^{-4}(\text{PA}) - 1.177 \times 10^{-6}(\text{PA})^2] \text{T} \\ &\quad + [1.001 \times 10^{-4} - 7.046 \times 10^{-7}(\text{PA}) + 1.355 \times 10^{-8}(\text{PA})^2] \text{T}^2 \\ \text{GR} &= [1.0 \times 10^{-5}] + [-1.9687 + 4.209 \times 10^{-1}(\text{GW}) + 3.9445 \times 10^{-2}(\text{GW})^2] \text{TOF} \\ \text{FGR} &= \text{GR} - [8.363 \times 10^{-2} + 1.488 \times 10^{-2}(\text{GR}) - 9.78 \times 10^{-5}(\text{GR})^2] \text{WS} \end{aligned}$$

### 4. CARGO

#### C-5

$$\begin{aligned} \text{TOF} &= [4.65478 + 6.94444 \times 10^{-3}(\text{T})] + [3.257 \times 10^{-1} + 2.7778 \times 10^{-4}(\text{T})] (\text{PA}/10) \\ \text{GR} &= [.1457 + 3.5625 \times 10^{-2}(\text{GW}) - 6.763 \times 10^{-5}(\text{GW})^2] \\ &\quad + [5.1428 - 3.175 \times 10^{-2}(\text{GW}) + 7.0089 \times 10^{-5}(\text{GW})^2] \text{TOF} \\ \text{FGR} &= \text{GR} - [0.1 + .0082(\text{GR})] \text{WS} \end{aligned}$$

C-7

$$\begin{aligned}\text{TOF} &= [7.90371 + 6.68965 \times 10^{-2}(\text{PA}) + 2.12622 \times 10^{-4}(\text{PA})^2] \\ &+ [3.00808 \times 10^{-2} + 2.67118 \times 10^{-5}(\text{PA}) + 9.85 \times 10^{-6}(\text{PA})^2]T \\ &+ [1.23149 \times 10^{-4} + 1.3589 \times 10^{-6}(\text{PA}) - 3.1641 \times 10^{-8}(\text{PA})^2]T^2 \\ \text{GR} &= [2.1742857 + 2.04286 \times 10^{-1}(\text{GW}) - 1.071429 \times 10^{-2}(\text{GW})^2] \\ &+ [1.14943 - 1.2707 \times 10^{-1}(\text{GW}) + 5.1785 \times 10^{-3}(\text{GW})^2]\text{TOF} \\ \text{FGR} &= \text{GR} - [-2.7327 \times 10^{-2} + 1.904 \times 10^{-2}(\text{GR})]\text{WS} \\ &+ [-6.308077 \times 10^{-4} + 1.94654 \times 10^{-4}(\text{GR})]\text{WS}^2\end{aligned}$$

C-9

$$\begin{aligned}\text{TOF} &= [1.2192956 + 2.2091577 \times 10^{-3}(\text{PA}) + 3.380102 \times 10^{-4}(\text{PA})^2] \\ &+ [1.4628966 \times 10^{-2} + 2.6313968 \times 10^{-4}(\text{PA}) - 1.3818053 \times 10^{-7}(\text{PA})^2]T \\ &- [2.4891 \times 10^{-4} - 6.875 \times 10^{-6}(\text{PA}) + 7.8125 \times 10^{-8}(\text{PA})^2]T^2 \\ &+ [2.20314 \times 10^{-6} - 6.49 \times 10^{-8}(\text{PA}) + 7.47 \times 10^{-10}(\text{PA})^2]T^3 \\ \text{GR}' &= [2.3806396 - 5.9265772 \times 10^{-2}(\text{GW}) + 6.67969 \times 10^{-4}(\text{GW})^2] \\ &+ [-1.19933136 + 5.041098 \times 10^{-2}(\text{GW}) - 2.12517 \times 10^{-4}(\text{GW})^2]\text{TOF} \\ \text{GR} &= \text{GR}' \times 10 \\ \text{FGR}' &= [1.0 + 9.7757143 \times 10^1(\text{GR}) + 6.4285714 \times 10^{-2}(\text{GR})^2] \\ &- [4.8785706 + 5.4275515 \times 10^{-1}(\text{GR}) + 4.438775 \times 10^{-3}(\text{GR})^2]\text{WS} \\ \text{FGR} &= \text{FGR}'/100\end{aligned}$$

C-130

$$\begin{aligned}
\text{TOF} &= [-4.799107 \times 10^{-1} + 3.3165178 \times 10^{-2}(\text{PA}) + 2.7902 \times 10^{-4}(\text{PA})^2] \\
&+ [2.129 \times 10^{-2} + 2.2538 \times 10^{-4}(\text{PA}) - 2.9186 \times 10^{-6}(\text{PA})^2] \text{T} \\
\text{GR} &= [1.16103 + 5.318 \times 10^{-2}(\text{GW}) + 9.0525 \times 10^{-4}(\text{GW})^2] \\
&+ [3.3695 \times 10^1 - 6.94278 \times 10^{-1}(\text{GW}) + 3.8559 \times 10^{-3}(\text{GW})^2] \text{TOF} \\
&- [-9.041 + 2.307 \times 10^{-1}(\text{GW}) - 1.264 \times 10^{-3}(\text{GW})^2] \text{TOF}^2 \\
&+ [-1.0708 + 2.477 \times 10^{-2}(\text{GW}) - 1.108 \times 10^{-4}(\text{GW})^2] \text{TOF}^3 \\
\text{FGR} &= \text{GR} - [2.4131 \times 10^{-1} + 2.115 \times 10^{-4}(\text{GR}) + 1.935 \times 10^{-4}(\text{GR})^2] \text{WS}
\end{aligned}$$

C-135

$$\begin{aligned}
\text{TOF} &= [3.9116 \times 10^{-2} + 6.3976 \times 10^{-2}(\text{PA})] + [1.6557 \times 10^{-2} - 7.6643 \times 10^{-6}(\text{PA})] \text{T} \\
\text{GR} &= [5.625 - 9.5 \times 10^{-2}(\text{GW}) + 1.3125 \times 10^{-3}(\text{GW})^2] \\
&+ [8.6496 \times 10^{-1} - 1.2768 \times 10^{-2}(\text{GW}) + 1.077 \times 10^{-4}(\text{GW})^2] \text{TOF} \\
&+ [4.0067 \times 10^{-1} - 5.982 \times 10^{-3}(\text{GW}) + 3.627 \times 10^{-5}(\text{GW})^2] \text{TOF}^2 \\
\text{FGR} &= \text{GR} - [1.508 \times 10^{-1} + 8.625 \times 10^{-3}(\text{GR})] \text{WS}
\end{aligned}$$

C-141

$$\begin{aligned}
\text{TOF} &= [5.4067 \times 10^1 - 1.3375 \times 10^{-1}(\text{PA}) - 2.2755 \times 10^{-4}(\text{PA})^2 + 3.6508 \times 10^{-6}(\text{PA})^3] \\
&- [7.395 \times 10^{-2} - 1.71 \times 10^{-4}(\text{PA}) - 5.91 \times 10^{-6}(\text{PA})^2 + 4.22 \times 10^{-8}(\text{PA})^3] \text{T} \\
\text{GR} &= [8.6549 \times 10^3 - 7.75196 \times 10^1(\text{GW}) + 2.07846 \times 10^{-1}(\text{GW})^2] \\
&- [5.6302 \times 10^2 - 4.9948(\text{GW}) + 1.30519 \times 10^{-2}(\text{GW})^2] \text{TOF} \\
&+ [1.22509 \times 10^1 - 1.07805 \times 10^{-1}(\text{GW}) + 2.759985 \times 10^{-4}(\text{GW})^2] \text{TOF}^2 \\
&- [8.8948 \times 10^{-2} - 7.77463 \times 10^{-4}(\text{GW}) + 1.956483 \times 10^{-6}(\text{GW})^2] \text{TOF}^3 \\
\text{FGR} &= \text{GR} - [1.4123219 \times 10^{-1} + 8.5293578 \times 10^{-3}(\text{GR}) + 5.709895 \times 10^{-6}(\text{GR})^2] \text{WS}
\end{aligned}$$

## 5. TRAINING

### T-29

$$\begin{aligned}\text{TOF} &= [7.83935 \times 10^{-1} + 5.38189 \times 10^{-2}(\text{PA})] \\ &+ [1.20408 \times 10^{-2} + 9.888357 \times 10^{-5}(\text{PA}) - 2.32448 \times 10^{-6}(\text{PA})^2]T \\ &- [9.72 \times 10^{-6} + 1.8278 \times 10^{-6}(\text{PA}) - 2.405 \times 10^{-8}(\text{PA})^2]T^2 \\ \text{GR} &= [3.18978 \times 10^1 - 1.785(\text{GW}) + 3.602 \times 10^{-2}(\text{GW})^2] \\ &+ [-8.8285 + 5.1387 \times 10^{-1}(\text{GW}) - 5.679 \times 10^{-3}(\text{GW})^2]_{\text{TOF}} \\ &+ [-1.76441 + 4.82709 \times 10^{-2}(\text{GW})]_{\text{TOF}}^2 \\ \text{FGR} &= \text{GR} - [8.6457 \times 10^{-2} + 1.1414 \times 10^{-2}(\text{GR})]_{\text{WS}}\end{aligned}$$

### T-33

$$\begin{aligned}\text{TOF} &= [-2.890514 \times 10^{-1} + 5.8370956 \times 10^{-2}(\text{PA})] \\ &+ [4.161561 \times 10^{-2} - 3.518445 \times 10^{-5}(\text{PA})]T \\ &+ [-6.0515 \times 10^{-5} + 3.53095 \times 10^{-6}(\text{PA})]T^2 \\ \text{GR} &= [-2.684337 \times 10^1 + 3.224954(\text{GW})] + [-2.0581519 + 3.7024356 \times 10^{-1}(\text{GW})]_{\text{TOF}} \\ &+ [-8.861357 \times 10^{-1} + 8.3093188 \times 10^{-2}(\text{GW})]_{\text{TOF}}^2 \\ \text{FGR} &= \text{GR} - [1.3583333 \times 10^{-1} + 9.5833 \times 10^{-3}(\text{GR})]_{\text{WS}}\end{aligned}$$

### T-37

$$\begin{aligned}\text{TOF} &= [7.46275 \times 10^{-1} + 1.789924 \times 10^{-2}(\text{PA}) + 1.667729 \times 10^{-4}(\text{PA})^2] \\ &+ [6.1017875 \times 10^{-3} + 3.4816947 \times 10^{-4}(\text{PA}) - 1.6406229 \times 10^{-6}(\text{PA})^2]T \\ &+ [1.718525 \times 10^{-4} - 2.621825 \times 10^{-6}(\text{PA}) + 4.184375 \times 10^{-8}(\text{PA})^2]T^2 \\ \text{GR} &= [-7.2378129 \times 10^1 + 3.8485684 \times 10^1(\text{GW}) - 6.565(\text{GW})^2 + 3.916 \times 10^{-1}(\text{GW})^3] \\ &+ [-5.477 \times 10^1 + 2.92 \times 10^1(\text{GW}) - 4.975(\text{GW})^2 + 2.906 \times 10^{-1}(\text{GW})^3]_{\text{TOF}} \\ \text{FGR} &= [-1.607758 + 1.222176(\text{GR}) - 5.64375 \times 10^{-3}(\text{GR})^2] \\ &- [.482382 \times 10^{-1} + 2.2260152 \times 10^{-2}(\text{GR}) - 4.7462116 \times 10^{-4}(\text{GR})^2]_{\text{WS}}\end{aligned}$$

T-38

$$\begin{aligned} \text{TOF} &= [1.996 + 1.69 \times 10^{-2}(\text{PA}) + 2.56 \times 10^{-5}(\text{PA})^2] \\ &+ [8.64 \times 10^{-3} - 7.5 \times 10^{-5}(\text{PA}) + 1.61 \times 10^{-6}(\text{PA})^2] \text{T} \\ \text{GR} &= [6.26 \times 10^1 - 1.299 \times 10^1(\text{GW}) + 6.886 \times 10^{-1}(\text{GW})^2] \\ &+ [-1.0004 \times 10^2 + 2.0317 \times 10^1(\text{GW}) - 9.67 \times 10^{-1}(\text{GW})^2] \text{TOF} \\ &+ [1.30368 \times 10^1 - 2.689(\text{GW}) + 1.403 \times 10^{-1}(\text{GW})^2] \text{TOF}^2 \\ \text{FGR} &= [-3.3 \times 10^{-1} + 1.047(\text{GR}) - 8.57 \times 10^{-4}(\text{GR})^2] \\ &- [4.22 \times 10^{-2} + 9.47 \times 10^{-3}(\text{GR}) + 1.9898 \times 10^{-5}(\text{GR})^2] \text{WS} \end{aligned}$$

T-39

$$\begin{aligned} \text{TOF} &= [6.6742857 \times 10^{-1} + 4.4226786 \times 10^{-2}(\text{PA})] \\ &+ [1.027143 \times 10^{-2} + 3.051339 \times 10^{-4}(\text{PA})] \text{T} \\ &+ [1.74994 \times 10^{-4} + 5.023 \times 10^{-7}(\text{PA})] \text{T}^2 \\ \text{GR} &= [-1.37666666 \times 10^1 + 1.67916666(\text{GW})] + [-3.55 + 4.71875 \times 10^{-1}(\text{GW})] \text{TOF} \\ \text{FGR} &= \text{GR} - [1.516666666 \times 10^{-1} + 1.008333333 \times 10^{-2}(\text{GR})] \text{WS} \end{aligned}$$

6. OBSERVATION

0-2

$$\begin{aligned} \text{TOF} &= [-9.2083337 \times 10^{-1} + 5.9113889 \times 10^{-2}(\text{PA})] \\ &+ [2.291666 \times 10^{-2} - 2.7778 \times 10^{-5}(\text{PA})] \text{T} \\ \text{GR} &= [3.711176 \times 10^1 - 1.640279 \times 10^1(\text{GW}) + 2.22809(\text{GW})^2] \\ &+ [-2.09922 \times 10^1 + 8.6991796(\text{GW}) - 8.4586 \times 10^{-1}(\text{GW})^2] \text{TOF} \\ &+ [2.248949 - 9.093486 \times 10^{-1}(\text{GW}) + 1.061975 \times 10^{-1}(\text{GW})^2] \text{TOF}^2 \\ \text{FGR} &= \text{GR} - [4.3358 \times 10^{-2} + 2.196 \times 10^{-2}(\text{GR})] \text{WS} \\ &+ [8.79209 \times 10^{-4} + 8.21219 \times 10^{-5}(\text{GR})] \text{WS}^2 \end{aligned}$$

OV-10

$$\begin{aligned} \text{TOF} = & [-6.46 \times 10^{-1} + 6.7857 \times 10^{-2}(\text{PA}) + 2.723 \times 10^{-4}(\text{PA})^2] \\ & + [3.69 \times 10^{-2} - 2.24 \times 10^{-3}(\text{PA}) + 3.49 \times 10^{-5}(\text{PA})^2]T \\ & + [1.07 \times 10^{-4} + 3.85 \times 10^{-5}(\text{PA}) - 4.688 \times 10^{-7}(\text{PA})^2]T^2 \end{aligned}$$

$$\begin{aligned} \text{GR} = & [5.38 - 1.105(\text{GW}) + 1.14 \times 10^{-1}(\text{GW})^2] \\ & + [8.02 \times 10^{-1} - 2.57 \times 10^{-1}(\text{GW}) + 2.4 \times 10^{-2}(\text{GW})^2] \text{TOF} \end{aligned}$$

$$\text{FGR} = \text{GR} - [1.6 \times 10^{-2} + 2.44 \times 10^{-2}(\text{GR}) - 2.128 \times 10^{-4}(\text{GR})^2] \text{WS}$$

## APPENDIX B

### TYPICAL TAKEOFF LENGTHS FOR ALL USAF AIRCRAFT

The takeoff length equations were used to determine the takeoff lengths for major USAF aircraft at a temperature of 60°F, pressure altitude of 3000 feet, a headwind velocity of 3 knots, and the average gross weight of the aircraft. The aircraft are divided into six categories: Bombers, Fighters, Attack, Cargo, Training, and Observation and are presented below:

<u>AIRCRAFT</u>	<u>GROSS WEIGHT (1000 lbs)</u>	<u>TAKEOFF LENGTH (ft)</u>
1. BOMBERS		
B-52	340	6000
B-57	45	4330
2. FIGHTERS		
F-4	50	3350
F-5	18	5790
F-100	36	7180
F-101	45	3190
F-102	30	2850
F-104	20	4380
F-105	45	4260
F-106	35	3960
F-111	75	3190
3. ATTACK		
A-7	30	3480
A-37	11	2300
4. CARGO		
C-5	520	4920
C-7	24	1420
C-9	84	6250
C-130	100	1740
C-135 (KC-135)	220	6300
C-141	220	2610
5. TRAINING		
T-29	50	4480
T-33	14	3410
T-37	6	1800
T-38	14	2570
T-39	14	2038
6. OBSERVATION		
O-2	5	1430
OV-10	11	1040



## APPENDIX C

### EXAMPLE OF APPLIED CURVE FITTING TECHNIQUE

A sample takeoff length equation has been calculated using the procedure outlined in section II, subsection 2 and the data contained in figure 1, 2, and 3. The least-squares equations utilized and the sample calculations are presented below (ref. 1).

#### 1. EQUATIONS TO CALCULATE COEFFICIENTS

a. Least-squares curve fitting techniques were applied to data taken from curves in figures 1, 2, and 3. The dependence of one variable on another was mathematically determined and expressed in the linear form

$$Y = m_1X + m_0$$

$Y$  = the dependent variable

$X$  = the independent variable

$m_1$  = first order coefficient

$m_0$  = a constant

or the second order form

$$Y = n_2X^2 + n_1X + n_0$$

where  $Y$  = the dependent variable

$X$  = the independent variable

$n_2$  = second order coefficient

$n_1$  = first order coefficient

$n_0$  = a constant

The coefficients  $m_1$  and  $m_0$  can be calculated from an equation in the form

- 
1. Volk, William, Applied Statistics for Engineers, McGraw-Hill Book Company, Inc., New York, 1958, pp. 224-295.

$$m_1 = \frac{\sum (X - \bar{X})(Y - \bar{Y})}{\sum (X - \bar{X})^2}$$

$$m_0 = \bar{Y} - m_1 \bar{X}$$

$$\text{where } \bar{Y} = \frac{\sum Y}{N}$$

$$\bar{X} = \frac{\sum X}{N}$$

N = Number of Values

The coefficients  $n_2$ ,  $n_1$  and  $n_0$  can be calculated from an equation in the form

$$n_2 = \frac{\begin{bmatrix} \sum (Y - \bar{Y})(X^2 - \bar{X}^2) \end{bmatrix} \begin{bmatrix} \sum (X - \bar{X})^2 \end{bmatrix} - \begin{bmatrix} \sum (Y - \bar{Y})(X - \bar{X}) \end{bmatrix} \begin{bmatrix} \sum (X - \bar{X})(X^2 - \bar{X}^2) \end{bmatrix}}{\begin{bmatrix} \sum (X - \bar{X})^2 \end{bmatrix} \begin{bmatrix} \sum (X^2 - \bar{X}^2)^2 \end{bmatrix} - \begin{bmatrix} \sum (X - \bar{X})(X^2 - \bar{X}^2) \end{bmatrix}^2}$$

$$n_1 = \frac{\begin{bmatrix} \sum (Y - \bar{Y})(X - \bar{X}) \end{bmatrix} \begin{bmatrix} \sum (X^2 - \bar{X}^2)^2 \end{bmatrix} - \begin{bmatrix} \sum (Y - \bar{Y})(X^2 - \bar{X}^2) \end{bmatrix} \begin{bmatrix} \sum (X - \bar{X})(X^2 - \bar{X}^2) \end{bmatrix}}{\begin{bmatrix} \sum (X - \bar{X})^2 \end{bmatrix} \begin{bmatrix} \sum (X^2 - \bar{X}^2)^2 \end{bmatrix} - \begin{bmatrix} \sum (X - \bar{X})(X^2 - \bar{X}^2) \end{bmatrix}^2}$$

$$n_0 = \bar{Y} - n_1 \bar{X} - n_2 \bar{X}^2$$

$$\text{where } \bar{Y} = \frac{\sum Y}{N}$$

$$\bar{X} = \frac{\sum X}{N}$$

$$\bar{X}^2 = \frac{\sum X^2}{N}$$

N = Number of Values

## 2. SAMPLE CALCULATION

a. Data to determine an equation for takeoff factor (TOF) as a function of temperature (T) was taken from the curves in figure 1. The lines are straight and, therefore, only X and Y values for the endpoints of the line are needed to determine the coefficients. For example, to determine an equation of takeoff factor versus temperature for pressure altitude (PA) of 2000 ft. the following data was taken from figure 1 and used to calculate the coefficients  $m_1$  and  $m_0$  in a linear equation.

<u>Y Value</u>	<u>X Value</u>
2.95	$1.0 \times 10^1$
4.00	$1.1 \times 10^2$

The result is an equation in the form of

$$\text{TOF} = 1.05 \times 10^{-2}T + 2.845$$

Since this equation is linear and was determined from two points, there is no difference between the calculated and actual values.

b. This method can be applied to all five lines in figure 1 and result in five equations, each corresponding to a different pressure altitude.

$$(\text{PA}=2000) \quad \text{TOF} = 1.05 \times 10^{-2}T + 2.845$$

$$(\text{PA}=4000) \quad \text{TOF} = 1.60 \times 10^{-2}T + 3.540$$

$$(\text{PA}=6000) \quad \text{TOF} = 2.05 \times 10^{-2}T + 4.145$$

$$(\text{PA}=8000) \quad \text{TOF} = 2.35 \times 10^{-2}T + 4.715$$

$$(\text{PA}=10000) \quad \text{TOF} = 2.50 \times 10^{-2}T + 5.250$$

c. The coefficients of the above equations can be mathematically related to pressure altitude. Since the coefficients change non-linearly with respect to pressure altitude, a second order equation will be developed. The following data was used to determine an equation for the first coefficient as a function of pressure altitude. The values for pressure altitude are divided by 100.

<u>Y Value</u>	<u>X Value</u>
$1.05 \times 10^{-2}$	$2.0 \times 10^1$
$1.60 \times 10^{-2}$	$4.0 \times 10^1$
$2.05 \times 10^{-2}$	$6.0 \times 10^1$
$2.35 \times 10^{-2}$	$8.0 \times 10^1$
$2.50 \times 10^{-2}$	$1.0 \times 10^2$

The resulting equation is in the form

$$m_1 = - 1.45833 \times 10^{-6} (PA)^2 + 3.625 \times 10^{-4} (PA) + 3.8333 \times 10^{-3}$$

Similarly,  $m_0$  is calculated and is in the form

$$m_0 = - 8.541667 \times 10^{-5} (PA)^2 + 3.96250 \times 10^{-2} (PA) + 2.091667$$

Both equations are found to yield calculated values to well within 10 percent of the actual values. By substitution, the result is an equation in the form

$$\begin{aligned} TOF = & [-1.45833 \times 10^{-6} (PA)^2 + 3.625 \times 10^{-4} (PA) + 3.8333 \times 10^{-3}] T \\ & + [-8.541667 \times 10^{-5} (PA)^2 + 3.9625 \times 10^{-2} (PA) + 2.091667] \end{aligned}$$

where PA = Pressure Altitude in hundreds of feet

T = Temperature in F°

This equation was tested and found to be accurate to within 10 percent.

d. The above method can be applied to data in figure 2. This will result in a group of equations which relate ground run and takeoff factor for five different gross weights.

$$(GW=90,000) \quad GR = 4.37575 \times 10^{-1} (TOF)^2 - 1.37575 (TOF) + 1.5 \times 10^1$$

$$(GW=110,000) \quad GR = 4.046568 \times 10^{-1} (TOF)^2 - 4.465676 \times 10^{-1} (TOF) + 1.75 \times 10^1$$

$$(GW=130,000) \quad GR = 3.799679 \times 10^{-1} (TOF)^2 + 2.503212 \times 10^{-1} (TOF) + 2.0 \times 10^1$$

$$(GW=150,000) \quad GR = 3.635086 \times 10^{-1} (TOF)^2 + 7.149137 \times 10^{-1} (TOF) + 2.25 \times 10^1$$

$$(GW=170,000) \quad GR = 3.55279 \times 10^{-1} (TOF)^2 + 9.4721 \times 10^{-1} (TOF) + 2.5 \times 10^1$$

e. As before, the coefficients of the above equations can be related as a function of gross weight. The first and second coefficients were found to be related by a second order equation. The third coefficient changed linearly with gross weight and required only a first order equation. The final equation which related ground run as a function of takeoff factor and gross weight is in the form

$$\begin{aligned} GR = & [1.01875 \times 10^{-5}(GW)^2 - 3.6813 \times 10^{-3}(GW) + 6.86394 \times 10^{-1}]TOF^2 \\ & + [-2.90437 \times 10^{-4}(GW)^2 + 1.04554 \times 10^{-1}(GW) - 8.43329]TOF \\ & + [1.25 \times 10^{-1}(GW) + 3.75] \end{aligned}$$

where GW = Gross Weight of the aircraft in thousands of pounds

GR = Ground Run in hundreds of feet

f. By an analagous method the true or final ground run can be related as a function of ground run and wind speed. This equation is in the form

$$FGR = GR - [5.875 \times 10^{-3}(GR) + 8.25 \times 10^{-2}]WS$$

where FGR = Final Ground Run in hundreds of feet

WS = Headwind Speed in Knots

The total equation is in the form

$$\begin{aligned} TOF = & [-1.45833 \times 10^{-6}(PA)^2 + 3.625 \times 10^{-4}(PA) + 3.8333 \times 10^{-3}]T \\ & + [-8.541667 \times 10^{-5}(PA)^2 + 3.9625 \times 10^{-2}(PA) + 2.091667] \\ GR = & [1.01875 \times 10^{-5}(GW)^2 - 3.6813 \times 10^{-3}(GW) + 6.8639 \times 10^{-1}]TOF^2 \\ & + [-2.90437 \times 10^{-4}(GW)^2 + 1.0455 \times 10^{-1}(GW) - 8.43329]TOF \\ & + [1.25 \times 10^{-1}(GW) + 3.75] \\ FGR = & GR - [5.875 \times 10^{-3}(GR) + 8.25 \times 10^{-2}]WS \end{aligned}$$

This equation was tested and found to be accurate to within 10 percent.

# APPENDIX D

## TAKEOFF CLIMB ANGLES FOR USAF AIRCRAFT

<u>AIRCRAFT</u>	<u>MAXIMUM CLIMB- ANGLE (deg)</u>	<u>MINIMUM CLIMB- ANGLE (deg)</u>	<u>VERTICAL FLIGHT DISTANCE (ft)*</u>
1. BOMBERS			
B-52	9	3	5000
B-57	17	6	4000
2. FIGHTERS			
F-4	47	21	5000
F-5	16	9	5000
F-100	22	8	5000
F-101	39	22	5000
F-102	15	7	5000
F-104	13	7	5000
F-105	26	8	5000
F-106	18	12	5000
F-111	22	9	5000
3. ATTACK			
A-7	12	8	5000
A-37	22	8	5000
4. CARGO			
C-5	13	3	3000
C-7	9	6	4000
C-9	8	5	3000
C-130	15	2	3000
C-135 (KC-135)	15	6	5000
C-141	13	5	5000
5. TRAINING			
T-29	19	5	3000
T-33	6	3	5000
T-37	11	7	5000
T-38	16	5	5000
T-39	15	5	5000
6. OBSERVATION			
O-2	8	4	4000
OV-10	16	4	5000

\*Angles are averaged from ground level to this altitude.